Classically improved method for the loading of probability distributions in quantum circuits

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1 Overview

The discrete nature of quantum computing requires for any probability distribution that needs to be loaded into a quantum state, to first be discretized. We develop a method to perform an optimal discretization, and show that it is suitable to be used on real quantum hardware, showing improvements with respect to existing packages.

2 Our method: optimal linear quantizers

The quantization of a random variable X consists of its approximation by a random variable Y taking finitely many values. The aim is to choose the random variable Y so as to minimize the resulting error for a fixed quantization level N. Optimal quantization is a variance reduction method that can be used to dramatically reduce the computation time of a Monte Carlo simulation or to increase its accuracy. We developed different families of optimal quantizations of a random variable, which we call optimal quantizers.

Among these, linear quantizers are equally spaced quantization points of X, and are most suitable for being encoded into a quantum computer. Specifically, we defined the quantizers as projections of X on a Voronoi partition of the relevant space $C = \{C_1, C_2, \ldots, C_N\}$,

$$Y := \mathrm{Proj}(X) = \sum_{i=1}^{N} y_i \mathbb{1}_{C_i}(X), \tag{1}$$

where y_i are the elements of a codebook. Optimal linear quantizers q_l are a discrete set of points defined by a discrete probability distribution $P(q_l)$.

3 Results

We apply an optimal linear quantizer method to classically obtain the best discretized normal probability distribution. The discretized points and corresponding probability values are such that the L^2 distance with respect to the standard normal distribution is minimised. This allows us, given a L^2 error, to use fewer qubits with respect to an arbitrary discretization in order to reach it.

The discretized probability distribution classically computed with optimal linear quantization is uploaded into the quantum circuit through a series of controlled rotations with suitable angles. We test and compare the quantum circuit to the `qiskit_finance` package "NormalDistribution" and as a result of a computation on `ibm_brisbane`'s real hardware QPUs (see Fig. 1). The results show that it is possible to obtain an improved approximation of the probability distribution with respect to this package, and that is easy to load on real quantum hardware.

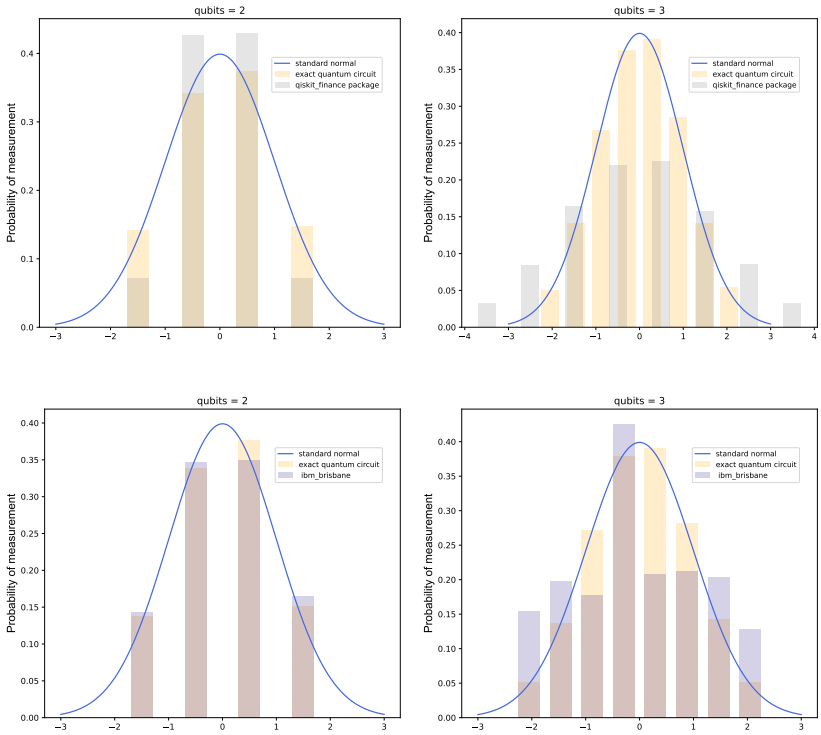


Figure 1: Output normal distribution for the exact noiseless quantum circuit (orange bars), compared with the `qiskit_finance` package output (grey bars) in top left and right figures, and with the output from `ibm_brisbane`'s real quantum hardware (purple bars) in lower left and right figures. The figures in the left column are computed on 2 qubits, the figures in the right column are computed on 3 qubits.